

BS3B

STATISTICAL LIFETIME-MODELS

David Steinsaltz – 16 lectures HT 2010
steinsal@stats.ox.ac.uk

Prerequisites

Part A Probability, Part A Statistics and Part B Applied Probability are prerequisites.

Website: <http://www.steinsaltz.me.uk/BS3b/BS3b.html>

Aims

Statistical Lifetime-Models follows on from Applied Probability. Models introduced there are examined in the first part of the course more specifically in a life insurance context where transitions typically model the passage from ‘alive’ to ‘dead’, possibly with intermediate stages like ‘loss of a limb’ or ‘critically ill’. The aim is to develop statistical methods to estimate transition rates and more specifically to construct life tables that form the basis in the calculation of life insurance premiums.

We will then move on to survival analysis, which is widely used in medical research, in addition to insurance, in which we consider the effect of covariates and of partially observed data. We also explain demographic concepts, and how life tables are adapted to the context of changing mortality rates.

Synopsis

Survival models: general lifetime distributions, force of mortality (hazard rate), survival function, specific mortality laws, the single decrement model, curtate lifetimes, life tables, period and cohort.

Estimation procedures for lifetime distributions: empirical lifetime distributions, censoring, Kaplan-Meier estimate, Nelson-Aalen estimate. Parametric models, accelerated life models including Weibull, log-normal, log-logistic. Plot-based methods for model selection. Proportional hazards, partial likelihood, semiparametric estimation of survival functions, use and overuse of proportional hazards in insurance calculations and epidemiology.

Two-state and multiple-state Markov models, with simplifying assumptions. Estimation of Markovian transition rates: Maximum likelihood estimators, time-varying transition rates, census approximation. Applications to reliability, medical statistics, ecology.

Graduation, including fitting Gompertz-Makeham model, comparison with standard life table: tests including chi-square test and grouping of signs test, serial correlations test; smoothness.

Demographic projection, Stable population age structures. Application to pension plans.

Exercises and Classes

Classes will be held on Fridays. There will be four sessions: 10-11, 11-12, 2-3, 3-4. Class assignments will be available on Minerva at <https://minerva.stats.ox.ac.uk>.

Scripts are to be handed in by Mondays 4pm in the Department of Statistics.

The scope of exercises goes significantly beyond that of exam questions in many cases, but understanding the exercises is essential to coping with the variety of exam questions that might come up. There is a great range of difficulty in the exercises, and most students should find at least some of the exercises very challenging. Try all of them, but don't spend hours and hours on questions if you are not making any progress.

Try to start solving exercises when you get the problem sheet, not the day before you have to hand in your solutions. This allows you to have second attempts at exercises that you can't solve straight away.

Lecture notes are meant to be useful when solving exercises. You may use any result from the lectures, except where the contrary is explicitly stated.

Reading

There are lots of good books on survival analysis. Look for one that suits you. Some pointers will be given in the lecture notes to readings that are connected, but look in the index to find topics that confuse you and/or interest you.

The actuarial material in the course is modeled on the CT4 Core Reading from the Institute of Actuaries.

CT4 Models Core Reading. Faculty & Institute of Actuaries

This is the core reading for the actuarial professional examination on survival models. In some places, the approach is more practically oriented and often placed in an insurance context, whereas the course is more academic and not only oriented towards insurance applications. All in all, this is the main reference for about half the course. It is available for about £21.50 from the Institute of Actuaries on Worcester Street. (A few college libraries have it.)

D.R. Cox and D. Oakes: *Analysis of Survival Data*. Chapman & Hall (1984)

This is *the* classical text on survival analysis. The presentation is concise, but gives a broad view of the subject. The text contains exercises. This is the main reference for about half the course. It contains also much more related material beyond the scope of the course.

H.U. Gerber: *Life Insurance Mathematics*. 3rd edition, Springer (1997)

The presentation is concise. Only three chapters are relevant. Chapter 2 gives an introduction to lifetime distributions, Chapter 7 discusses the multiple decrement model and Chapter 11 estimation procedures for lifetime distributions. The remainder combines the ideas with the interest rate theory of BS4.

Klein & Moeschberger: *Survival Analysis: Techniques for Censored and Truncated Data*, 2nd edition, Springer (2003)

This is an excellent source for a lot of the survival analysis topics, particularly censoring and truncation, and the Kaplan-Meier and Nelson-Aalen estimators. Lots of terrific examples.