

Exercise Sheet Lecture 6 — The Normal distribution

1. If $X \sim N(200, 20^2)$, calculate
 - (a) $P(X < 210)$
 - (b) $P(X < 195)$
 - (c) $P(X > 215)$
 - (d) $P(185 < X < 205)$
 - (e) a such that $P(X > a) = 0.2$
2. Accidents on a certain railway line occur at an average rate of one every 2 months. Find the probability that
 - (a) there are 25 or more accidents in 4 years,
 - (b) there are 30 or fewer accidents in 5 years.
3. In a certain country the heights of adult males have mean 170cm and standard deviation 10cm, and the heights of adult females have mean 160cm and standard deviation 8cm; for each sex the distribution of heights approximates closely to a normal probability model. On the hypothesis that height is not a factor in selecting a mate, calculate the probability that
 - (a) a husband and wife selected at random are both taller than 164cm
 - (b) in a randomly selected husband and wife the wife is taller than the husband
 - (c) the average height of a random couple is greater than 168cm.
4. 10% of chocolates produced in a factory are mis-shapes. In a sample of 1000 chocolates find the probability that the number of mis-shapes is
 - (a) less than 80
 - (b) between 90 and 115 inclusive
 - (c) 120 or more.

5. A sample of 100 apples is taken from a load. The apples have the following distribution of sizes

| | | | | | |
|------------------------|----|----|----|----|----|
| Diameter to nearest cm | 6 | 7 | 8 | 9 | 10 |
| Frequency | 11 | 21 | 38 | 17 | 13 |

Determine the mean and standard deviation of these diameters.

Assuming that the distribution is approximately normal with the estimated mean and standard deviation, find the range of size of apples for packing, if 5% are to be rejected as too small and 5% are to be rejected as too large.

6. The median weight of 2-year-old boys in the UK is 12.1 kg. The 75th percentile (3rd quartile) is 13.05 kg. Assuming the distribution of weights follows the normal distribution, what fraction of 2-year-old boys are heavier than 14.5 kg?
7. The figure below shows 143 genetics students from the University of Connecticut, arranged by height. (The figure first appeared in the *Hartford Courant* in 1996, and is discussed in [SWW02]. The photo was downloaded from mindprod.com/jgloss/histogram.html.) The women are wearing white, the men black.

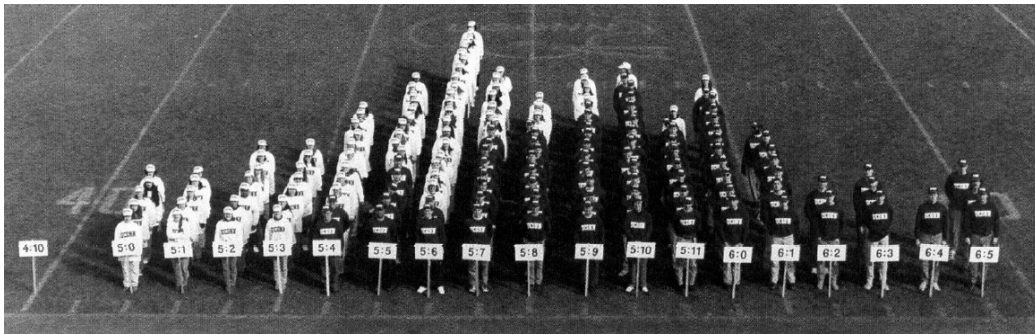


Figure 1: The living histogram.

- (a) Draw a histogram of all the students, of the men separately, and of the women separately.
- (b) Which of these would you expect to look about like a normal curve? Do they look like normal curves? If there is a histogram which ought to look normal and doesn't, can you think of a plausible explanation for why it doesn't?

- (c) Suppose men's and women's heights have SD σ_{MEN} and σ_{WOMEN} . Let σ_{MIX} be the SD of the whole population. Which would you expect to be true? a) σ_{MIX} between σ_{MEN} and σ_{WOMEN} ; b) σ_{MIX} substantially larger than σ_{MEN} and σ_{WOMEN} ; c) σ_{MIX} substantially smaller than σ_{MEN} and σ_{WOMEN} .

References

- [SWW02] Mark F. Schilling, Ann E. Watkins, and William Watkins. Is human height bimodal? *The American Statistician*, 56(3):223–9, 2002.