

**Probability and Statistics for Psychology
and Quantitative Methods for Human Sciences**
Problem Sheet 1: Confidence Intervals and Normal Approximation

1. A study was carried out to test the prevalence of side effects from the pertussis vaccine.¹ Of 339 infants who received their first injection of vaccine, 69 showed adverse reactions.
 - (a) Compute 95% and 99% confidence intervals for the probability of an adverse reaction to the vaccine.
 - (b) What do these confidence intervals mean? What assumptions go into the interpretation?

2. Michaelson and Morley performed a famous experiment in 1887 to measure the speed of light with extraordinary precision. The values measured in 100 trials are given in the table below. (Speeds are given as the number of km/sec above 299,000. Thus, the first measurement represents a measurement of 299,850 km/sec.) The measurements are normally distributed.

850	740	900	1070	930	850	950	980	980	880
1000	980	930	650	760	810	1000	1000	960	960
960	940	960	940	880	800	850	880	900	840
830	790	810	880	880	830	800	790	760	800
880	880	880	860	720	720	620	860	970	950
880	910	850	870	840	840	850	840	840	840
890	810	810	820	800	770	760	740	750	760
910	920	890	860	880	720	840	850	850	780
890	840	780	810	760	810	790	810	820	850
870	870	810	740	810	940	950	800	810	870

- (a) Assuming that the measurement errors were normally distributed with mean 0, compute a 99.9% confidence interval for the true speed of light.
 - (b) The currently accepted value of the speed of light is 299,734.5 km/sec. What conclusion can you draw?

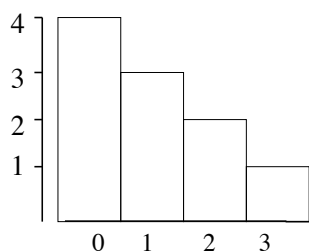
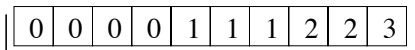
3. 100 students each perform an experiment to estimate a parameter μ , and each one independently computes a 99% confidence interval for μ . What is the probability that there will be at least 3 students whose confidence intervals do not include μ ? (Hint: Use the binomial distribution or the Poisson distribution.)

4. Here are data on house prices, for all house sales in 2007 in the county of Hypotheshire: median = £200,000, mean = £255,000, SD = £145,000. There were 6112 house sales altogether.
 - (a) Would it be reasonable to infer that about 68% of all house sale prices in the city were in the range £110,000 – £400,000? Explain.

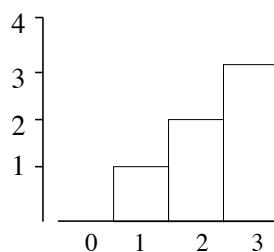
¹Published as “Whooping-cough vaccination: An assessment,” Miller et al., *The Lancet*, 1974. Described in Samuels and Witmer, *Statistics for Life Sciences*, p.212.

(b) Can you compute a 95% confidence interval for the average house sale price in 2007? Why or why not?

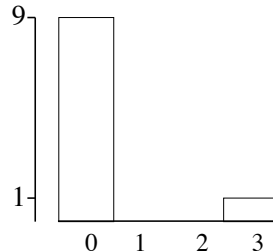
5. 1000 draws are made at random (with replacement) from the box with 10 numbers in it



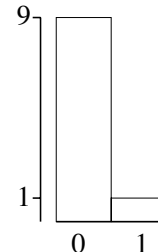
(a)



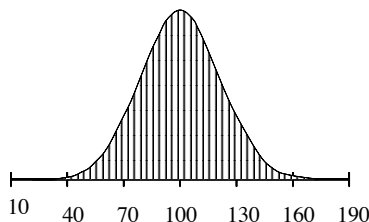
(b)



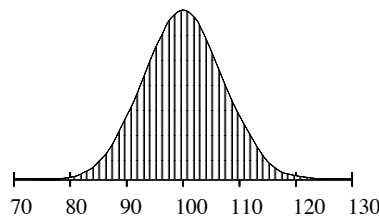
(c)



(d)



(e)



(f)

The vertical scales in these graphs are different, but each has total area 100%. In statements (i)–(iv) indicate whether the histogram described should look exactly or approximately like which one of the given histograms, or whether it does not resemble any of them. (A probability histogram is a histogram representing a probability distribution, with bars over x having a height proportional to the probability of getting x .)

- (a) A histogram of the 1000 numbers drawn will look EXACTLY / APPROXIMATELY like a b c d e f.
- (b) The probability histogram of the first draw looks EXACTLY / APPROXIMATELY like a b c d e f.
- (c) The probability histogram for the sum of the first 100 draws looks EXACTLY/ APPROXIMATELY like a b c d e f.
- (d) The probability histogram for the number of times 3 is drawn in all 1000 draws looks EXACTLY / APPROXIMATELY like a b c d e f.
- (e) Find the approximate probability that 2 is drawn 25 or more times among the 100 draws.
- (f) Find the approximate probability that the sum of the 100 draws is exactly 100.