

Exercise Sheet Lecture 5 — The Poisson distribution

1. State the conditions under which the Poisson distribution can be used to approximate a Binomial distribution?

If $X \sim \text{Bin}(200, 0.01)$, calculate $P(X \leq 2)$

- (a) using a Binomial distribution
 - (b) using a Poisson approximation to the Binomial
2. The mean number of bacteria per milliliter of a liquid is known to be 4. Assuming that the number of bacteria follows a Poisson distribution, find the probability that, in 1ml of liquid, there will be
 - (a) no bacteria
 - (b) 4 bacteria
 - (c) fewer than 3 bacteria

Find the probability that

- (a) in 3ml of liquid there will be less than 2 bacteria
 - (b) in 0.5ml of liquid there will be more than 2 bacteria
3. Stroke patients with aphasic deficits are each given a number of straight-forward tasks in a psychometric test. The number of errors made by 123 patients are shown in the table below. Calculate the mean and variance of the number of errors per patient and comment on these values. Fit a Poisson distribution and comment on how well it fits the observed data.

Number of errors	0	1	2	3	4	5 or more
Number of patients	5	30	56	15	10	7

4. In a large town, one person in 80, on average, has blood type X. If 200 blood donors are taken at random, find an approximation to the probability that they include at least five persons having blood type X.

How many donors must be taken at random in order that the probability of including at least one donor of type X shall be 0.9 or more?

5. Telephone calls reach a secretary independently and at random, internal ones at a mean rate of 2 in any 5 minute period, and external ones at a mean rate of 1 in any 5 minute period. Calculate the probability that there will be more than 2 calls in any period of 2 minutes.