

## Exercise Sheet Lecture 8 — The $\chi^2$ test

1. The following table gives the observed counts in 1-second intervals of alpha particles emitted from a radioactive source. Use a  $\chi^2$  test at the 5% level to assess the goodness of fit to the Poisson distribution.

$n$	Observed
0	5267
1	4436
2	1800
3	534
4	111
5+	21

2. Taken from [POD08, section 12.1]. It is hypothesised that when homing pigeons are disoriented in a certain manner, they will fly off in a completely random direction. To test this, 120 pigeons are disoriented, let loose, and the direction of flight of each is recorded. The following are the resulting data:

Direction	# Pigeons	Direction	# Pigeons
0–45°	12	180–225°	13
45–90°	16	225–270°	20
90–135°	17	270–315°	17
135–180°	15	315–360°	10

Formulate the chance model for the null hypothesis explicitly, and perform an appropriate test at the 0.01 significance level. What conclusions can you draw?

3. The paper [Coc36] looked at a study of 1440 tomato plants in 24 rows of 60 plants each. Each plant was classified as healthy or diseased (affected by spotted wilt). In all, 261 plants were diseased. The question was raised, whether the disease is randomly distributed through the field. To investigate this, the field was divided into 160 blocks of 9 plants each (3 rows by 3 plants), and the number of diseased plants (a number 0 through 9) in each block was counted. The frequency of each of the 10 possibilities is enumerated below.

$x$	0	1	2	3	4	5	6	7	8	9
Frequency	36	48	38	23	10	3	1	1	0	0

If the diseased plants are uniformly distributed, the number of diseased plants in a random block is binomially distributed. Fit a Binomial distribution to the data. Group small classes appropriately, and assess the fit using a  $\chi^2$  test at the 1% level. State clearly the null hypothesis; what do you think would be a reasonable alternative hypothesis?

4. Over a period of time a mens' clothes shop sold 500 pairs of trousers. The numbers sold in various waist sizes were:

Waist (cm)	70	75	80	85	90	95	100	105	110
Frequency	5	13	37	88	114	123	82	29	9

Estimate the mean and standard deviation of the waist measurements, and test whether the figures support the theory that waist measurements follow a normal distribution. Use a 5% level of significance. (Note that a customer with, for example, a waist measurement between 80 and 85cm requires trousers with an 85cm waistband.)

5. Responding to anecdotal reports in air squadrons that military pilots father more girls than boys, [Sny61] tabulated the sex of pilots' offspring for three kinds of flight duty during the month of conception. Is there any evidence of an association between father's activity and offspring gender? Use a 5% level of significance.

	Female Offspring	Male Offspring
Flying Fighters	51	38
Flying Transports	14	16
Not Flying	38	46

*(This example is taken from [Ric95], p. 500.)*

**Exam Paper Questions** The table below lists past exam paper *question numbers* that involve Chi-squared tests (useful for revision). NB. bracketed question numbers indicate that only part of the question involves Chi-squared tests.

	Human Sciences	Psychology		
Year	TT	MT	HT	TT
2001	3 7		2 3	3 7
2000	3 4 8	2 8	3 4	3 4 8
1999	3 7	3 7	(4) 10	3 7
1998	3 6		3 6	3 6
1997	5 7		3 7	5 7

## References

- [Coc36] William G. Cochran. The statistical analysis of field counts of diseased plants. *Supplement to the Journal of the Royal Statistical Society*, 3(1):49–67, 1936.
- [POD08] Roxy Peck, Chris Olsen, and Jay L. Devore. *Introduction to Statistics and Data Analysis*. Brooks/Cole, Belmont, CA, 3rd edition, 2008.
- [Ric95] John A. Rice. *Mathematical Statistics and Data Analysis*. Duxbury Press, 1995.
- [Sny61] Richard G. Snyder. The sex ratio of offspring of pilots of high performance military aircraft. *Human Biology*, 33(1):1–10, 1961.